MATH 115 FINAL EXAMINATION

This is an open-book exam. You may refer to your text and other course materials as you work on the exam, and you may use a calculator. **You must complete the exam individually. Neither collaboration nor consultation with others is allowed.**

Record your answers and work on the separate answer sheet provided.

There are 28 problems.
Problems #1–6 are Multiple Choice.
Problems #7–17 are Short Answer. (Work not required to be shown)
Problems #18–28 are Short Answer with work required to be shown.

MULTIPLE CHOICE

1. Solve $|7 - 4x| \geq 5$ and write interval notation for the solution set. 1. _____

A. [1/2, 3]
B. [1/2, ∞)
C. (–∞, 3] ∪ [1/2, ∞)
D. (–∞, 1/2] ∪ [3, ∞)

2. Which of the following polynomials has a graph which exhibits the end behavior of downward to the left and upward to the right? 2. _____

A. $f(x) = 4x^5 + x + 8$
B. $f(x) = -5x^4 - 9x^2 - 5$
C. $f(x) = -2x^3 - 3x^3 - x$
D. $f(x) = 3x^6 + 7x^3 - x - 5$

3. Write as an equivalent expression: $3 \log (x + 2) + \log 1 - \log y$ 3. _____

A. $\log\left(\dfrac{(x+2)^3}{y}\right)$

B. $\log\left(\dfrac{x^3+8}{y}\right)$

C. $\dfrac{6\log(x)+1}{\log y}$

D. $\log(3x+7-y)$

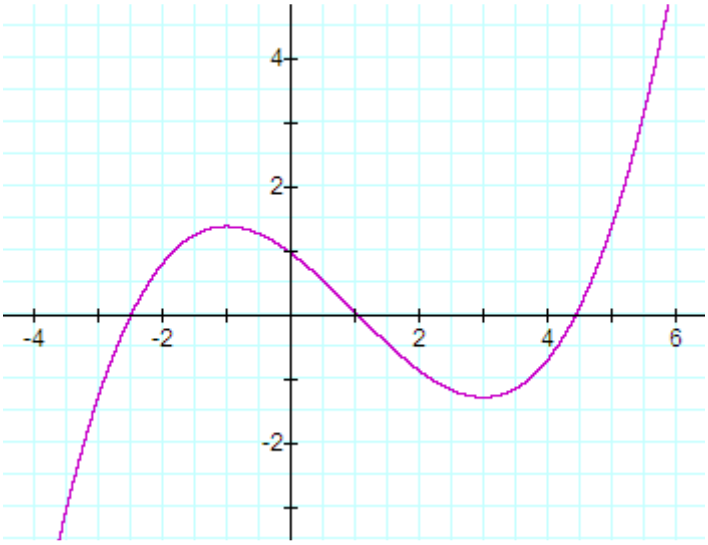
4. Determine the interval(s) on which the function is decreasing. 4. _____

A. $(-\infty, -2.5)$ and $(1, 4.5)$

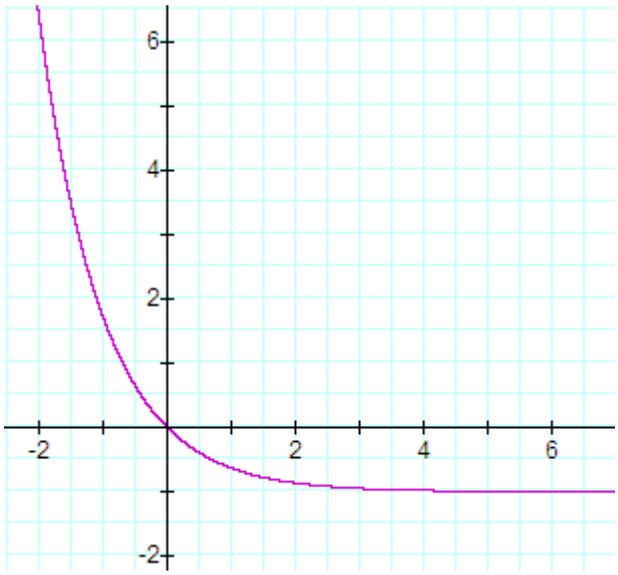
B. $(-\infty, -1)$ and $(3, \infty)$

C. $(-1, 3)$

D. $(-1.3, 1.3)$



5. Which of the functions corresponds to the graph? 5. _____



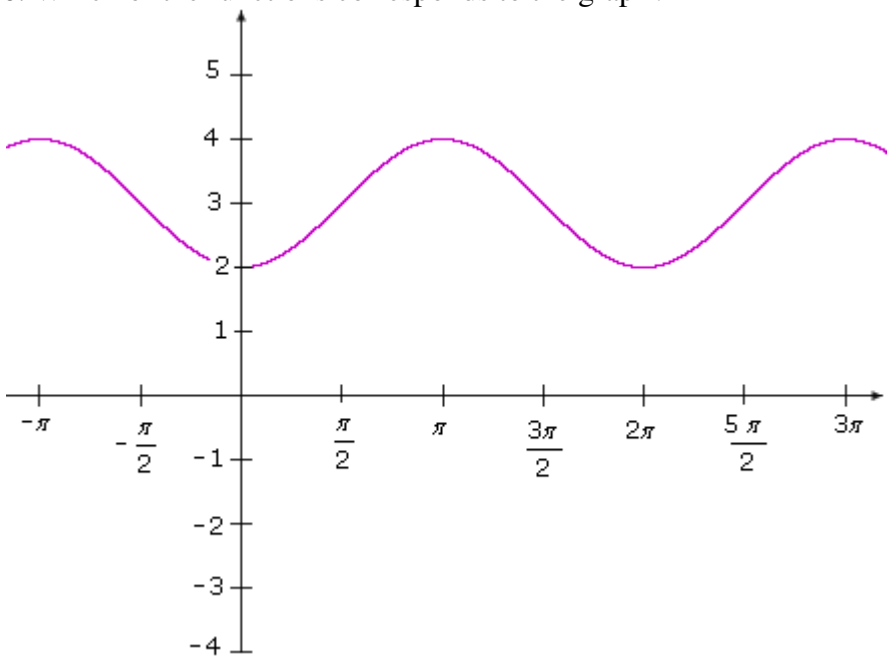
A. $f(x) = -e^x$

B. $f(x) = e^x - 2$

C. $f(x) = e^{-x} - 1$

D. $f(x) = e^{-x} + 1$

6. Which of the functions corresponds to the graph? 6. _____



A. $f(x) = 2 + \cos x$

B. $f(x) = 3 - \cos x$

C. $f(x) = 2(1 - \cos x)$

D. $f(x) = 2 - \sin x$

SHORT ANSWER:

7. Points (7, 2) and (3, 8) are endpoints of the diameter of a circle.

(a) What is the exact length of the diameter? (Simplify as much as possible) Answer: _____

(b) What is the center of the circle? Answer: _____

(c) What is the equation of the circle? Answer: _____

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8. Find the value of the logarithm: $\log_6 \left(\frac{1}{36}\right)$. Answer: _____

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9. Al, a resident of Metropolis, pays Metropolis an annual tax of \$65 plus 1.4% of his annual income. If Al paid \$863 in tax, what was Al's income?

Answer: _____

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10. A can of soda at 82° F. is placed in a refrigerator that maintains a constant temperature of 35° F. The temperature T of the soda t minutes after it is placed in the refrigerator is given by

$$T(t) = 35 + 47\, e^{-0.058\, t}$$

Find the temperature of the soda 10 minutes after it is placed in the refrigerator. (Round to the nearest degree.)

Answer: _____

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11. Given the function $f(x) = \frac{5}{8} + \frac{1}{4}x$, find a formula for the inverse function.

Answer: _____

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12. (a) State the reference angle associated with 120°. Answer: _____

(b) Convert 225° to radians. Leave the answer in terms of π. Answer: _____

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13. Given $y = 5\sin(8x - \pi)$, state the

(a) period Answer: _____

(b) phase shift Answer: _____

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14. Solve the trigonometric equation $(\cos x)(2\sin x - 1) = 0$ in the interval [0, 360°).

Answer: _____

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15. (a) Find the exact value of $\arcsin\left(\sin\frac{2\pi}{3}\right)$ Answer: _____

(b) Find the exact value of $\arccos\left(\tan\frac{3\pi}{4}\right)$ Answer: _____

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16. For the parabola given by $(x - 3)^2 = 8(y + 8)$, find the following:

(a) direction parabola opens (to the left, right, up, or down) Answer: _____

(b) vertex Answer: _____

(c) focus Answer: _____

17. Let $f(x)=\dfrac{3x+1}{x-2}$.

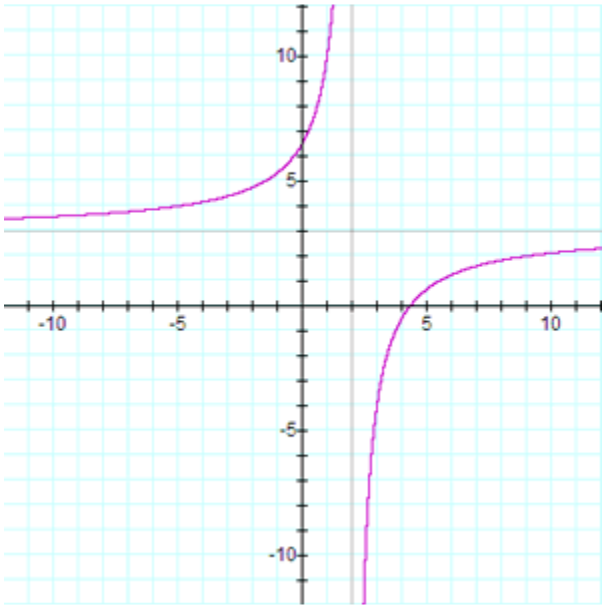
(a) State the domain. Answer: _____

(b) State the horizontal asymptote. Answer: _____

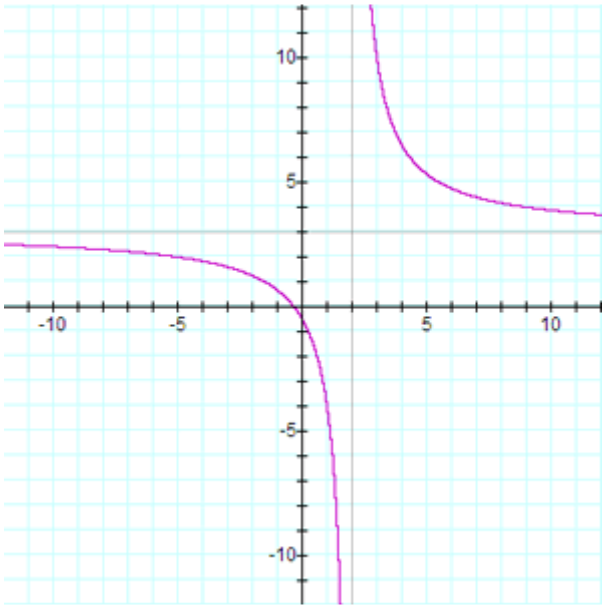
(c) State the vertical asymptote(s). Answer: _____

(d) Which of the following represents the graph of $f(x)=\dfrac{3x+1}{x-2}$? Answer: _____

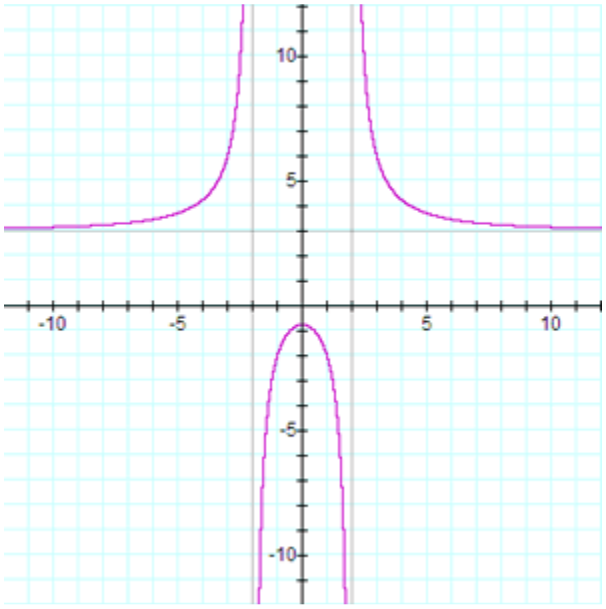
GRAPH A.



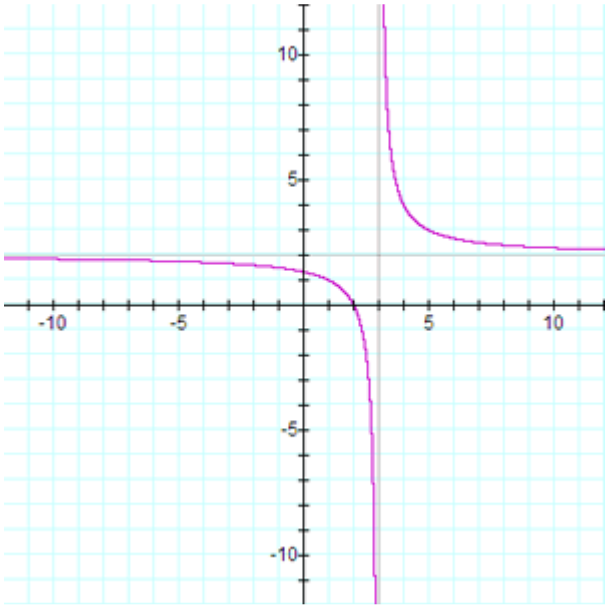
GRAPH B.



GRAPH C.



GRAPH D.



SHORT ANSWER, with work required to be shown, as indicated.

18. Find the equation for a line which passes through the points (5, 2) and (7, –8). Write the equation in slope-intercept form. **Show work.**

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19. Find the exact solutions and simplify as much as possible: $2x^2 = 7 + 4x$. **Show work.**

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20. Let $f(x) = 3x^2 - 4$ and $g(x) = x - 3$.

(a) Find the composite function $(f \circ g)(x)$ and simplify. **Show work.**
(b) Find $(f \circ g)(-2)$. **Show work.**

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21. A projectile is launched from a platform 12 feet high with an initial velocity of 72 feet per second, The height *h* of the projectile at *t* seconds after launch is given by $h = -16t^2 + 72t + 12$ feet.

(a) How many seconds after launch does the projectile attain maximum height? **Show work.**

(b) What is the maximum height? **Show work.**

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22. Solve: $\frac{x+7}{x+3} + \frac{24}{x^2-9} = 0$. **Show work.**

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23. Suppose that $\sin\theta = 4/5$ and that θ is a Quadrant II angle.

(a) Find the exact value of $\cos\theta$. **Show work**.

(b) Find the exact value of $\sin(2\theta)$. **Show work.**

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24. **Prove** the identity $(\cos x - \sin x)^2 + \sin(2x) = 1$

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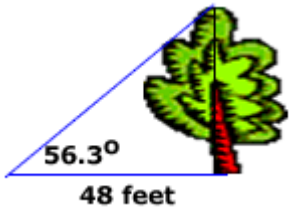
25. From a point 48 feet from the base of a redwood tree, the angle of elevation to the top of the tree is 56.3°. Find the height of the tree to the nearest foot. **Show work.**
(sketch is not to scale)



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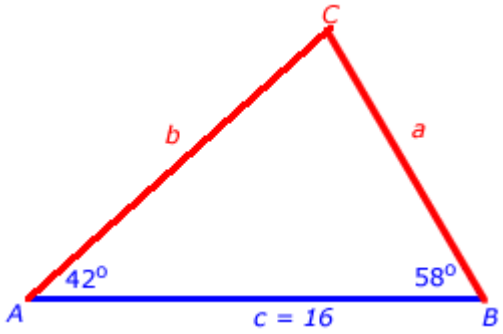
26. For the triangle *ABC*, we are given that $A = 42°$, $B = 58°$, and $c = 16.0$.



Find the length of side ***a***, rounded to the nearest tenth. **Show work.**

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27. Let $\vec{u} = \langle 16, 4 \rangle$ and $\vec{v} = \langle 2, -8 \rangle$.

(b) Calculate the dot product $\vec{u} \cdot \vec{v}$. **Show work.**

(c) Determine the angle between $\vec{u}$ and $\vec{v}$. Round the result to the nearest degree. **Show work.**

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28. An ellipse has the equation $\frac{(x-8)^2}{16} + \frac{(y+1)^2}{100} = 1$

(a) Is the major axis horizontal or vertical?

(b) Find the exact values of the foci of the ellipse. **Show work.**